Real-time proximal gradient method for linear MPC

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1 Introduction

Since the past few years, there has been an increased interest in using first-order methods for model predictive control (MPC). In contrast to active-set or interior-point schemes, first-order methods do not require the solution of a linear system at every iteration which makes them the ultimate choice to obtain fast MPC on resource-constrained embedded computing hardware [1]. This work focuses on the proximal gradient method (PGM) and proposes a PGM-based real-time iteration scheme for linear MPC.

2 Linear model predictive control

This work addresses the control of a discrete-time linear time-invariant (LTI) system of the form $x_{k+1} = Ax_k + Bu_k$. Given an estimate of the current state x_k, a linear MPC controller will compute an input trajectory $q = \{q(0), \ldots, q(N-1)\}$ over a time horizon of N samples by solving an optimal control problem of the form

$$\min_{q \in \mathscr{Q}(x_k)} V_N(x_k, q) = \frac{1}{2} q^T F q + \frac{1}{2} x_k^T G x_k + q^T H x_k \,, \qquad (1)$$

where $\mathscr{Q}(x_k)$ represents the set of feasible trajectories, including input limitations. An optimal MPC approach would then apply the first sample of the optimal solution $q^*(x_k)$ to the system, i.e. $u_k = q^*(x_k, 0)$.

3 Real-time proximal gradient method

The proximal gradient method (PGM) is an extension of the (projected) gradient method and a popular first-order method for linear MPC with simple input constraints. Applied to the OCP (1), the PGM iteratively updates the estimate for q as

$$q_k^+ = \Pi_{\mathscr{Q}(x_k)}(q_k - \gamma \nabla_q V_N(x_k, q_k)) \,, \qquad (2)$$

with γ the step size and $\Pi_{\mathscr{Q}(x_k)}(\cdot)$ the projection on $\mathscr{Q}(x_k)$.

This work proposes an MPC scheme that implements a real-time version of the PGM to solve OCP (1) in receding horizon. This means that only one PGM step (2) is performed per control update in stead of solving (1) to the desired accuracy. The first sample of the resulting input trajectory is applied to the system, i.e. $u_k = q_k^+(0)$, and a warm-start for the next update is computed using the current solution q_k^+.

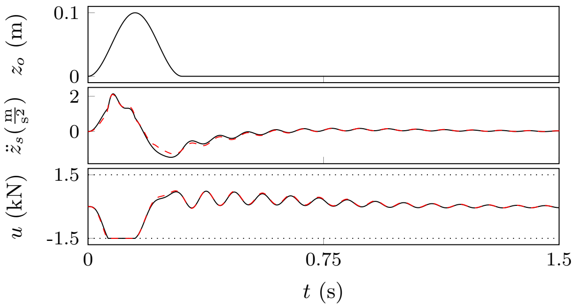

Figure 1: Bump response on a controlled quarter-car system. The black lines indicate the response using the real-time PGM while the dash red lines indicate the response using an optimal MPC approach.

When applying the real-time PGM scheme to LTI systems with simple input constraints, the resulting control law consists of simple steps involving matrix-vector multiplication, addition and saturation and offers possibilities to obtain fast control rates even on resource-constrained hardware such as PLCs or FPGAs. Closed-loop stability is proven for both the system's state and the input trajectory, i.e. over the control updates the system's state is attracted to a stable equilibrium while the suboptimal input trajectory is converging towards its optimal value.

4 Numerical example

The closed-loop performance of the real-time PGM is validated on a simulation example considering the control of an active vehicle suspension system based on a 2-DOF quarter-car model. The control objective is chosen to minimize the RMS acceleration of the car body. The resulting body acceleration $\ddot{z}_s$ and constrained control signal u are represented in Figure 1 when a bump road disturbance z_o is applied to the quarter-car system. The red dashed lines indicate the response using an optimal MPC approach. While the optimal MPC executes on average 100 PGM iterations during a control cycle, the real-time PGM only executes one iteration while the relative deviation in RMS body acceleration is limited to 0.04. One could thus conclude that the real-time PGM forms a computational cheap alternative for an optimal MPC approach with little loss in performance.

References

[1] J. L. Jerez, P. J. Goulart, S. Richter, G. A. Constantinides, E. C. Kerrigan, and M. Morari, "Embedded online optimization for model predictive control at megahertz rates," *IEEE Transactions on Automatic Control*, vol. 59, no. 12, pp. 3238–3251, 2014.

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